Supplemental materials for: Validation of a noisy Gaussian Boson Sampler via graph theory

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S1 Analysis of the graphs method for Borealis validation in the lossless scenario

The idea of the method is to arrange the output distributions of a gaussian boson sampler into the orbits. As described briefly in the main text, the orbits are the results of a coarse-graining of the output configurations. In this work, we considered the orbits of the first proposal in Ref.[50]. More precisely, given the number n of detected photons, we have the orbit $O_1 = [1, 1, 1, \cdots, 0]$ that collects the output states in which the number of photons in the modes is 0 or 1, $O_2 = [2, 1, 1, \cdots, 0]$ in which only one detector measures two photons, and $O_3 = [2, 2, 1, \cdots, 0]$ in which two modes host two photons. The various mock-up hypotheses differ for the *feature vectors*, i.e. the vectors that have the probability of the orbits as components. Such vectors have a strong connection with graphs theory for an ideal gaussian boson sampler with indistinguishable squeezed vacuum states in the inputs.

The distribution of the vectors in the space spanned by only two or three orbit components displays different behaviors according to the nature of the gaussian boson sampler. The method's theoretical proposal showed that collecting photon samples in the orbits from a small set of different unitary evolutions is enough to establish whether a classical model can emulate the experimental samples. The feature vectors extracted from the samples of a given model tend to cluster together by varying the unitary evolutions. Furthermore, the estimate of the orbit probability can be obtained efficiently from a finite sample with the required accuracy for the validation task. This is largely studied and demonstrated in Ref.[?] Then, it is possible to establish how far are the feature vectors extracted from the single-photon samples from the mock-up hypotheses by looking, for example, at the distance between the centroids of the clusters or the dispersion and shape of the clusters. For clarity, we report a practical example to explain the method's functioning and effectiveness.

In the simulation carried out in an ideal lossless condition, we consider the Borealis time-bin interferometer programmed to implement the 4 unitary transformations investigated in Fig.5b of the main text. Fig. **??** reports the distributions of the feature vectors of some mock-up hypotheses, like coherent and thermal light and distinguishable squeezed states. We also retrieved the feature vectors of a greedy sampler. All the data of the classical models are retrieved from sources that emit $n = 26$ photons on average, on a sample of 250000 events. At this point, we estimate the orbits probability for an ideal device with indistinguishable squeezed vacuum states. In the regime of the simulation, it is still possible to approximate the orbits probability (and so the ground truth) by a Monte Carlo method used in the theoretical proposal.[?] Since such a simulation shows that the orbit probabilities of an ideal device are very well separated from the others, we can use the distance of the centroid of experimental feature vectors for a given set of unitary evolution from the ones of the classical models as an effective quantity for the validation task. Furthermore, we require that such a distance should be larger than the typical dispersion of the cluster.

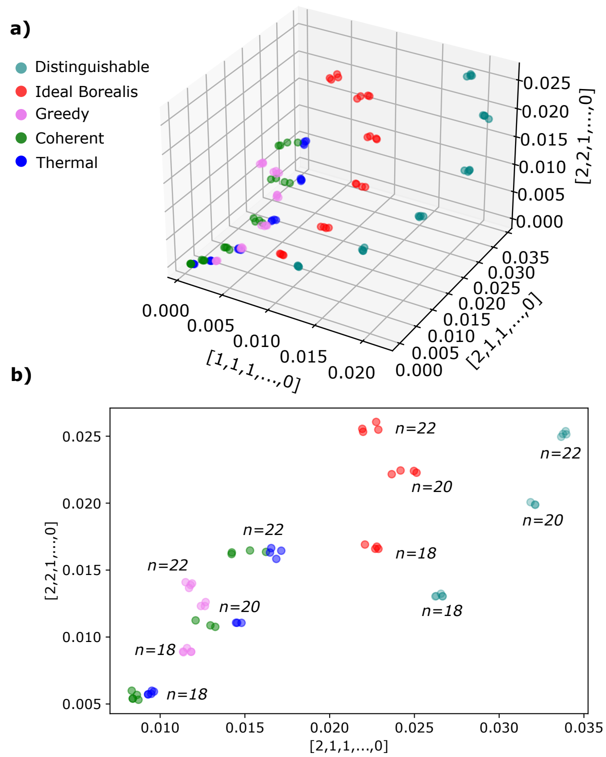

Figure S1 Validation though the graphs method. a) Distribution of the three orbits probabilities for the various mock-up hypotheses highlighted with different colors. The orbits of the mock-up hypotheses are retrieved from a sample of 250000 events generated by $m = 216$ gaussian states sources that emit on average $n = 26$ photons. The orbits correspond to 4 different configurations of the Borealis interfemoreter in a lossless condition. In red the ideal Borealis simulated data according to the method of Ref.[?] b) Same analysis with a focus on the plane spanned by O_2 and O_3 for a number of post-selected photons equal to $n = 18,\ 20,\ 22$.

S2 Simulation parameters and Borealis device certificate

Here we report a couple of the device certificates for Borealis. To be more precise, we report the device certificate for the 4th of April 2023, the day on which the measures for Fig. 5b were taken, and the device certificate for the 12th of January 2023, which is the base certificate for the thermal simulations of 4a.

The device certificate shows the 'loop_phases' that refers to the 3 static phases $\tilde{\phi}_{1,2,3}$ that cannot be controlled by the user and represent the optical phases of each loop. Then we have the 'squeezing_parameters_mean' which represents the squeezing values for the various presets. For most of our runs we employed the squeezer setting "low". The 'common_efficiency', 'loop_efficiencies', and 'relative_channel_efficiencies' are the 3 types of efficiencies that we discuss in Section 4 of the main text.

We use these parameters in the device certificate for the simulations. In the case of thermal, squashed, and coherent states simulations we generate the states so that in a lossless simulation the average number of photons is the same as that of the lossless simulation of the squeezed states. These values can be obtained directly from the squeezing value s. In the case of thermal and squashed states the mean photon-number population of the mode can be calculated as $\sinh^2(s)$ of the squeezing value, while for the coherent states, the displacement is calculated as $\sinh(s)$ of the squeezing value.

For the points of the thermal simulation in Fig. 4a, where we varied only the common efficiency, we started from the device certificate of the 12th of January, and we set the common efficiency to the values 0.350, 0.375, 0.400, 0.425 and 0.450, with the larger orbits corresponding to lower efficiencies. The green orbit, for which the detector number 5 was turned off to cause a significant unbalanced loss, was simulated with a common efficiency of 0.400. For the lossless indistinguishable SMSV states simulation we set the squeezing so that the average number of photons detected was 26.

This is the device certificate for the 4th of April 2023, used for Fig. 5b:


```
{'finished_at': '2023-04-04
  T13:51:32.915878+00:00',
 'target': 'borealis',
 'loop_phases': [1.281, 0.671, 0.472],
 'schmidt_number': 1.135,
 'common_efficiency': 0.361,
 'loop_efficiencies': [0.875,
 0.888, 0.717],
 'squeezing_parameters_mean':
 {'low': 0.704,
 'high': 1.178,
 'medium': 0.998},
 'relative_channel_efficiencies': [0.925,
  0.927, 0.915, 1.0, 0.965, 0.913,
  0.888, 0.968, 0.951, 0.946, 0.961,
  0.998, 0.952, 0.967, 0.946, 0.899]}
```


This is the device certificate for the 12th of January 2023, used for Fig. 4a:


```
{'finished_at': '2023-01-12
  T14:51:32.887242+00:00',
 'target': 'borealis',
 'loop_phases': [-0.797, 0.086, -2.689],
 'schmidt_number': 1.144,
 'common_efficiency': 0.392,
 'loop_efficiencies': [0.88,
 0.836, 0.734],
 'squeezing_parameters_mean':
 {'low': 0.669,
 'high': 1.149,
 'medium': 0.978},
 'relative_channel_efficiencies': [0.925,
  0.937, 0.914, 0.997, 0.978, 0.919,
  0.901, 0.971, 0.954, 0.956, 0.969,
  1.0, 0.942, 0.964, 0.96, 0.907]}
```

S3 Data from Borealis quantum advantage experiment

In the main text, we applied the method to some of the data from Ref.[5]. In particular, the various efficiencies and squeezing value are the following:

```
{'loop_phases': [0.67, 3.239, 3.516],
 'common_efficiency': 0.475,
 'loop_efficiencies': [0.915,
 0.895, 0.855],
 'squeezing_parameters_mean':
 {'low': 0.533}
 'relative_channel_efficiencies': [0.978,
 0.943, 0.958, 0.808, 0.924, 0.998,
 0.893, 0.893, 0.985, 1.0, 0.888,
 0.936, 0.973, 0.921, 0.883, 0.951]}
```

while the beam-splitter splitting ratios and phase shifters values were set at random in the ranges [0.4, 0.6] and $[-\frac{\pi}{2}, \frac{\pi}{2}]$ respectively.